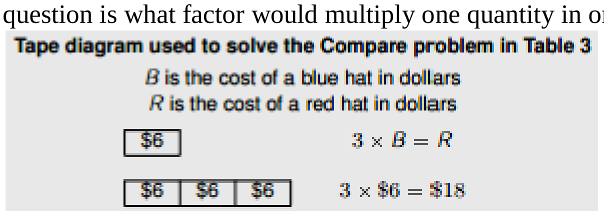
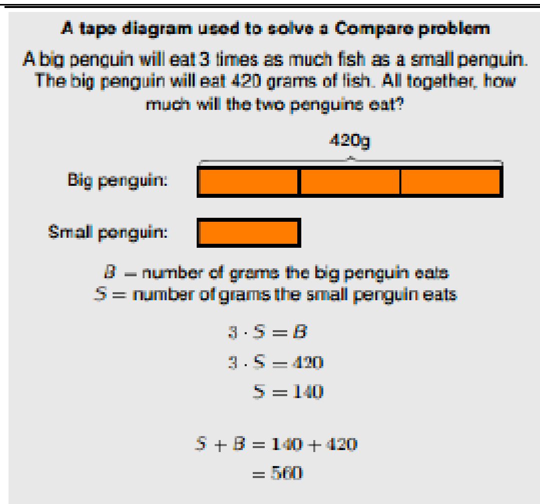
4th Grade

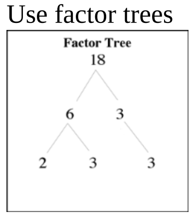
Instructional Focus:

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Standard	Objective	Examples
Operations and Algebraic Thinking		
Use the four operations with whole numbers to solve problems.		
4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 groups of 7 and 7 groups of 5 (Commutative property). Represent verbal statements of multiplicative comparisons as multiplication equations.	1. Demonstrate the commutative property of multiplication using models. 2. Apply the commutative property of multiplication with written and verbal equations.	Through the use of arrays, manipulatives, drawings, and the multiplication chart students will explore the commutative property of multiplication 5x7=35, 7x5=35 35= 5x7, 35=7x5 A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "a is n times as much as b"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given opportunities to write and identify equations and statements for multiplicative comparisons. 5 x 8 = 40. Sally is five years old. Her mom is eight times older. How old is Sally's Mom? 5 x 5 = 25 Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?
4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem or missing numbers in an array). Distinguish multiplicative	1. Multiply to solve word problems using drawings and equations with symbols for the unknown number. 2. Divide to solve word problems using drawings and equations with	This standard calls for students to translate comparative situations into equations with an unknown and solve. In an additive comparison, the underlying question is what amount would be added to one quantity in order to result in the other. In a multiplicative comparison, the underlying question is what factor would multiply one quantity in order to result in the other. 

comparison from additive comparison.	symbols for the unknown number. 3. Distinguish multiplicative comparison from additive comparison by noting that An. additive comparison focuses on the difference between two quantities (How many more.) b. multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other. (How many times as many.)	 (Progressions for the CCSSM: Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 29) Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p). Group Size Unknown: A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? (18 ÷ p = 3 or 3 x p = 18). Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18). When distinguishing multiplicative comparison from additive comparison, students should note that • additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, “How many more?” • multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is “How many times as much?” or “How many times as many?”
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including	1. Solve multistep word problems involving multiplication, division, addition, and subtraction. 2. Justify the reasonableness of an answer using mental computation estimation strategies including rounding.	The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations. On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? Some typical estimation strategies for this problem: -Student 1 I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. -Student 2 I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.

rounding.		-Student 3 I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30.When I added 300, 200 and 30, I know my answer will be about 530. The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected? Student 1 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles. Student 2 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300-60 = 240, so we need about 240 more bottles.
Gain familiarity with factors and multiples.		
4.OA.4. • Find all factor pairs for a whole number in the range 1–100. • Explain the correlation/differences between multiples and factors. • Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	1. Find all factor pairs for a whole number in the range 1–100. 2. Compare/ contrast multiples and factors. 3. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. 4. Explain whether a given whole number in the range 1–100 is prime or composite	 factor rainbow for 24 Multiples can be thought as the result of skip counting by each of the factors. Helpful hints: • All even numbers are multiples of 2 • All even numbers that can be halved twice are multiples of 4 • All numbers ending in 0 or 5 are multiples of 5 To find prime or composite numbers you can: • Use factor trees  • • Prime Sieve using Hundreds Chart • http://www.pedagonet.com/quickies/Eratosthenes.pdf.
Generate and analyze patterns.		
4.OA.5. Generate a number, shape pattern, table, t-chart, or input/output function that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule	1. Generate a number or shape pattern that follows a given rule. 2. Generate a table, t-chart or input/output function table using a given rule.	Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

<table>
<tr>
<td>itself. Be able to express the pattern in algebraic terms. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</td>
<td>3. Identify apparent features of the pattern that were not explicit in the rule itself.
4. Explain the patterns in algebraic terms.</td>
<td>


2, 4, 6, 8, 10….(even numbers: add 2 each time)

1, 4, 9, 16…. (squares)

2, 5, 11, 23 ….(double the number and add 1)

<table>
<tr><td>Pattern</td><td>Rule</td><td>Feature(s)</td></tr>
<tr><td>3, 8, 13, 18, 23, 28,</td><td>Start with 3, add 5</td><td>The numbers alternately end with a 3 or 8</td></tr>
<tr><td>5, 10, 15, 20 …</td><td>Start with 5, add 5</td><td>The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number.
The numbers that end in 0 are products of 5 and an even number.</td></tr>
</table>

Use the given rule to fill in the missing values.

1.

$y = x \div 5$	
Input (x)	Output (y)
15	
25	
20	
30	

2.

$y = 9x$	
Input (x)	Output (y)
4	
7	
2	
9	

Use the given rule to fill in the missing values.

1.

$y = x - 4$	
Input (x)	Output (y)
14	
11	
13	
5	

2.

Add 6	
Input (x)	Output (y)
8	
6	
4	
1	
</td>
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<tr>
<td>4.OA.6. Extend patterns that use addition, subtraction, multiplication, division or symbols, up to 10 terms, represented by models (function machines), tables, sequences, or in problem situations. (L)</td>
<td>1. Extend patterns that use addition or subtraction up to 10 terms, using models, tables, sequences, or in word problems.
2. Extend patterns that use multiplication or division up to 10 terms, using models, tables, sequences, or in word problems.
3. Extend patterns that use symbols up</td>
<td>
4,6,8,10…. List the next ten terms.

Marcy wants to go the movies with her friends. Each ticket costs $4.00. Make a table showing how much it would cost for each number of friends for up to five friends.

Num. of friends	Cost of ticket
1	$4.00
2	$8.00

Roberta is fishing on the Little Susitna River. She catches 3 salmon in one hour. In two hours, she catches 6 salmon. After three hours, she has caught 9 salmon. If this pattern continues, how many salmon will she have after 10 hours of fishing? Explain your answer using a visual model.

Lindsay is babysitting her neighbor for 4 hours. How much will she earn if she is paid $6 for each hour?
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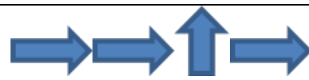
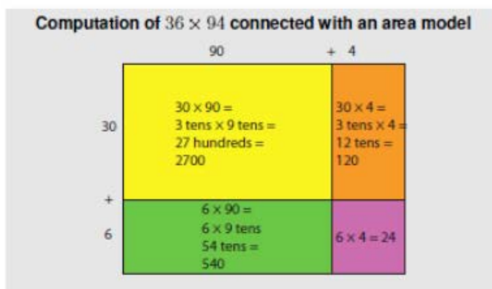
	to 10 terms, using models, tables, sequences, or in word problems.	 If this pattern repeats, fill in the next five symbols.
Number and Operations in Base Ten		
Generalize place value understanding for multi-digit whole numbers.		
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. Have students investigate the patterns associated with the answers obtained with calculators to problems such as the following. (They should relate their findings to the patterns on the place value chart.) 7 x 10 7 x 100 70 ÷ 10 7 x 1,000 700 ÷ 10 7 x 10,000 7,000 ÷ 10
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.	1. Read and write multi-digit whole numbers in base-ten numerals. 2. Read and write multi-digit whole numbers in number names. 3. Read and write multi-digit whole numbers in expanded form. 4. Compare multi-digit whole numbers using the symbols: <, >, = and record results.	This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form or number name is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand." The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system. Students should also be able to compare two multi-digit whole numbers using appropriate symbols.
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.	1. Round multi-digit whole numbers to any place using place value understanding. 2. Describe compare and contrast solutions using a variety of estimation methods.	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected? Student 1 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles. Student 2 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300-60 = 240, so we need about 240 more bottles.

		Example: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel? Some typical estimation strategies for this problem: Student 1 I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. Student 2 I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500. Student 3 I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530. Example: Round 368 to the nearest hundred. This will either be 300 or 400, since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400 
Use place value understanding and properties of operations to perform multi-digit arithmetic.		
4.NBT.4. Fluently add and subtract multi-digit whole numbers using any algorithm. Verify the reasonableness of the results.	1. Accurately and efficiently add multi-digit numbers using algorithms. 2. Accurately and efficiently subtract multi-digit numbers using algorithms. 3. Justify the reasonableness of the results of both addition and subtraction multi-digit numbers.	When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works. 3892 + 1567 1. Two ones plus seven ones is nine ones. 2. Nine tens plus six tens is 15 tens. 3. I am going to write down five tens and think of the10 tens as one more hundred.(notates with a 1 above the hundreds column) 4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds. 5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column) 6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand. 3546 - 928 Student justification for this problem: 1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.) 2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.) 3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.) 4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds. 5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer). 6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and	1. Multiply up to four-digit whole numbers by one digit whole number.	Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their

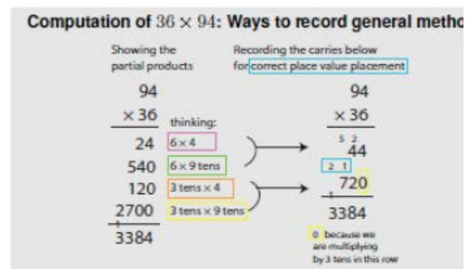
multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	2. Multiply two two-digit whole numbers using place value strategies. 3. Multiply two two-digit whole numbers using properties of operations. 4. Illustrate the calculations using equations, rectangular arrays, and area models. 5. Explain calculations through words and various models.	reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade. Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units.

Example:

$$\begin{aligned} 36 \times 94 &= (30+6) \times (90+4) \\ &= (30+6) \times 90 + (30+6) \times 4 \\ &= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4. \end{aligned}$$



The products of like base-ten units are shown as parts of a rectangular region.



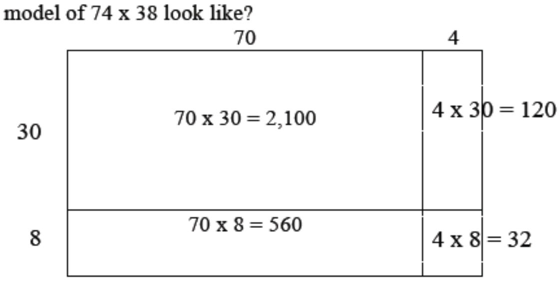
These proceed from right to left, but could go left to right. Or right, digits that represent newly composed tens and hundre are written below the line instead of above 94. The digits 2 a are surrounded by a blue box. The 1 from 30 × 4 = 120 is placed correctly in the hundreds place and the digit 2 from 30 × 90 = 2700 is placed correctly in the thousands place. these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow bo the ones place of the second line of the method on the right there because the whole line of digits is produced by multipl by 30 (not 3).

(Progressions for the CCSSM: Number and Operations Base Ten, CCSS Writing Team, May 2011, page 14)

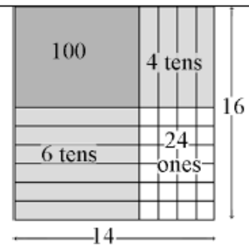
Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

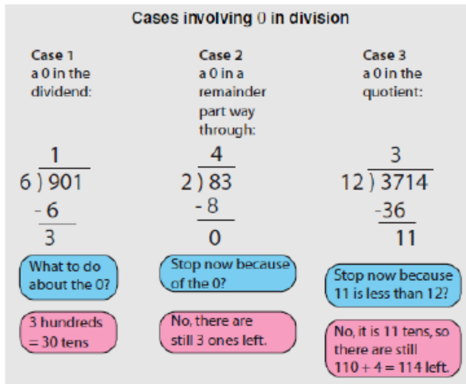
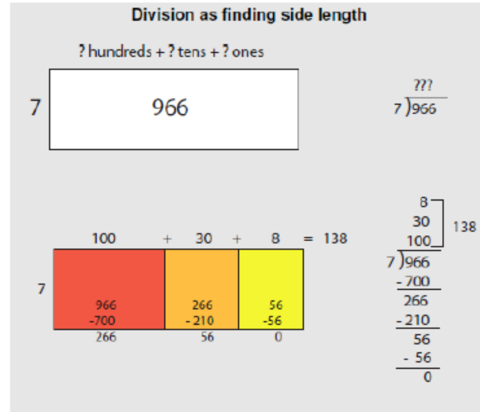
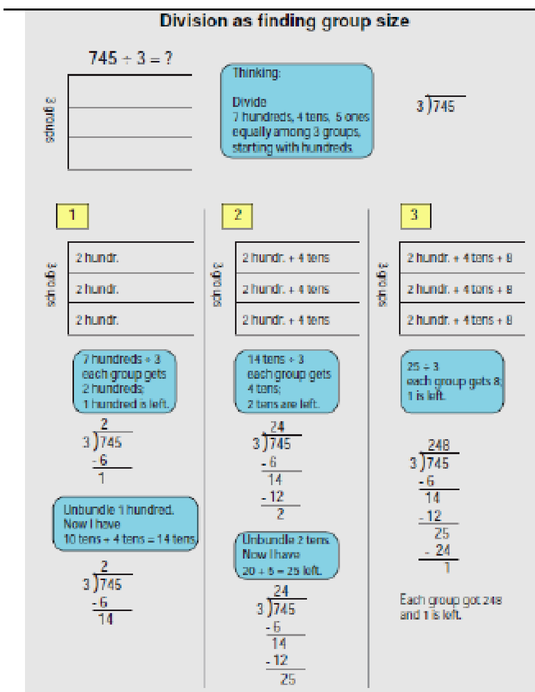
Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

Example:
What would an array area model of 74 x 38 look like?



2,100 = 560 + 1,200 + 32 = 2,812

		To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924. The area model below shows the partial products. 14 x 16 = 224  100 + 40 + 60 + 24 = 224 Using the area model, students first verbalize their understanding: • 10 x 10 is 100 • 4 x 10 is 40 • 10 x 6 is 60, and • 4 x 6 is 24. They use different strategies to record this type of thinking. Students explain this strategy and the one below with base 10 blocks, drawings, or numbers. 25 x24 400 (20 x 20) 100 (20 x 5) 80 (4 x 20) 20 (4 x 5) 600

4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	1. Solve whole-number quotients and remainders with up to four digit dividends and one-digit divisors based on place value, properties of operations and relationships between multiplication and division. 2. Illustrate whole-number quotients and remainders with up to four digit and one-digit divisors using place value strategies, rectangular arrays, area models and properties of operations. 3. Explain whole-number quotients and remainders with up to four digit dividends and one-digit divisors using place value strategies, rectangular arrays, area models and properties of operations.	  *966 ÷ 7 is viewed as finding the unknown side length of a rectangular region with area 966 square units and a side of length 7 units. The amount of hundreds is found, then tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, and 7 by 8. It can be connected with the decomposition of 966 as 7 × 100 + 7 × 30 + 7 × 8. By the distributive property, this is 7 × (100 + 30 + 8), so the unknown side length is 138. In the recording on the right, amounts of hundreds, tens, and ones are represented by numbers rather than by digits, e.g., 700 instead of 7.* (Progressions for the CCSSM: Numbers and Operations Base Ten, CCSS Writing Team, May 2011, page 14)  *745 ÷ 3 can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In Step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated, and the 1 is the number of hundreds not allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.* (Progressions for the CCSSM: Numbers and Operations Base Ten, CCSS Writing Team, May 2011, page 15) A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box? • Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

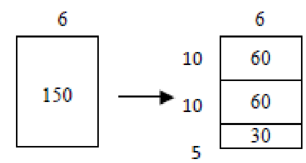
• Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
• Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

This standard calls for students to explore division through various strategies.

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

Student 1	Student 2		Student 3
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	592 -400 | 50 192 -160 | 20 32 -32 | 4 0	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

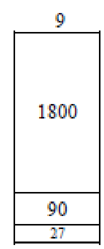
Example:
Using an Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.
Example: 150 ÷ 6



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

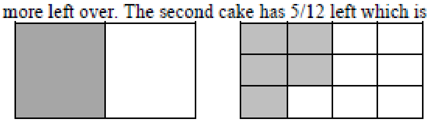
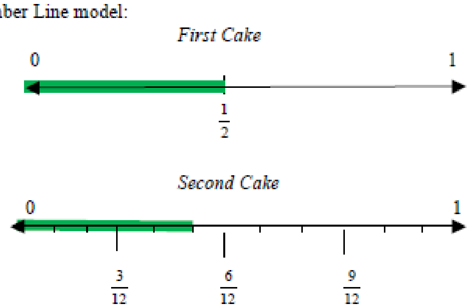
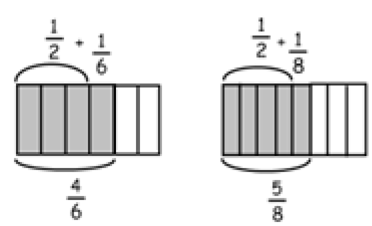
1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:
   a. 150   150 ÷ 6 = 10 + 10 + 5 = 25
      - 60 (6 x 10)
      90
      - 60 (6 x 10)
      30
      - 30 (6 x 5)
      0
   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

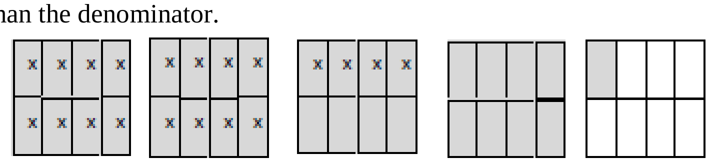
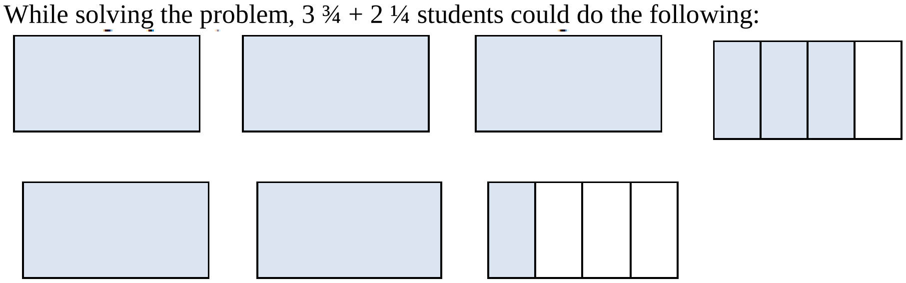
Example:
1917 ÷ 9



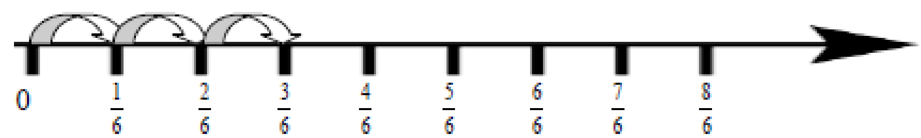
A student's description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can take 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.

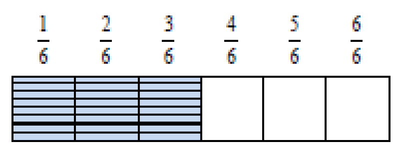
Numbers and Operations- Fractions		
Extend understanding of fraction equivalence and ordering.		
4.NF.1. Explain	1. Explain	Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*).

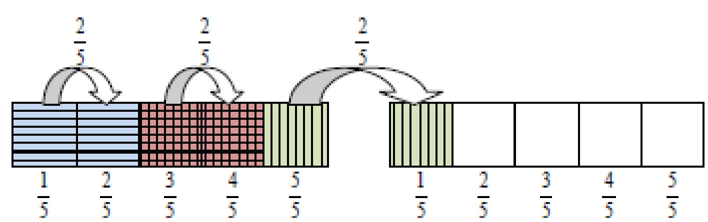
why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	equivalent fractions. 2. Demonstrate how the number and size of parts differ even though two fractions themselves are the same size using a visual fraction model. 3. Recognize equivalent fractions. 4. Generate equivalent fractions using visual models.	How do you know that 4/6 = 2/3? They are the same because you can simplify 4/6 and get 2/3. (procedural thinking) If you have a set of 6 things and you take 4 of them, that would be 4/6. You can make the 6 into groups of 2. So then there would be 3 groups, and the 4 would be 2 groups out of the 3 groups. That means it's 2/3. (conceptual thinking) If you start with 2/3, you can multiply the top and the bottom numbers by 2, and that will give you 4/6, so they are equal. (procedural thinking) If you had a square cut into 3 parts and you shaded 2, that would be 2/3 shaded. If you cut all 3 of these parts in half that would be 4 parts shaded and 6 parts in all. That's 4/6 and it would be the same amount.. (conceptual thinking)
4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	1. Compare two fractions with different numerators and different denominators by creating common numerators. 2. Compare two fractions with different numerators and different denominators by creating numerators. 3. Compare two fractions with different numerators and different denominators by comparing to a benchmark of 1/2. 4. Recognize that comparisons of fractions are valid only when referring to the same whole. 5. Record results of comparison of fractions with symbols >, <, =. 6. Justify and record results	Example: There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left? Student 1 Area model: The first cake has more left over. The second cake has 5/12 left which is smaller than ½.  Student 2 Number Line model:  Student 3 verbal explanation: I know that 6/12 equals ½. Therefore, the second cake which has 5/12 left is less than ½. Example: When using the benchmark of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:  $\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

	of fractional comparisons by using a visual fractional model.	(Progressions for the CCSSM: Numbers and Operations Fractions, CCSS Writing Team, May 2011, page 6)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.		
4.NF.3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). *For example: 3/8 = 1/8 + 1/8 + 1/8 ;3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators	a. 1. Understand and explain adding fractions as joining parts referring to the same whole. 2. Understand and explain subtraction of fractions as separating parts referring to the same whole. b. 1. Decompose a fraction into a sum of fractions with the same denominator in more than one way. 2. Decompose and record fractions into a sum of fractions with the same denominator using equations. 3. Justify decomposition of fractions with the same denominator using a visual fraction model. c. 1. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction. 2. Add mixed numbers with like denominators	a. The student will orally or in writing explain the following examples: 1. 2/3 + 1/3 = (1/3 + /3) + 1/3 2. 9/10—4/10 = 9/10—1/10—1/10– 1/10– 1/10 b 3/8=1/8 + 1/8 +1/8 3/8=1/8 + 2/8 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8 c. A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. Example: Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not. The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot. Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend? Possible solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas. Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator.  Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator . While solving the problem, 3 ¾ + 2 ¼ students could do the following: 

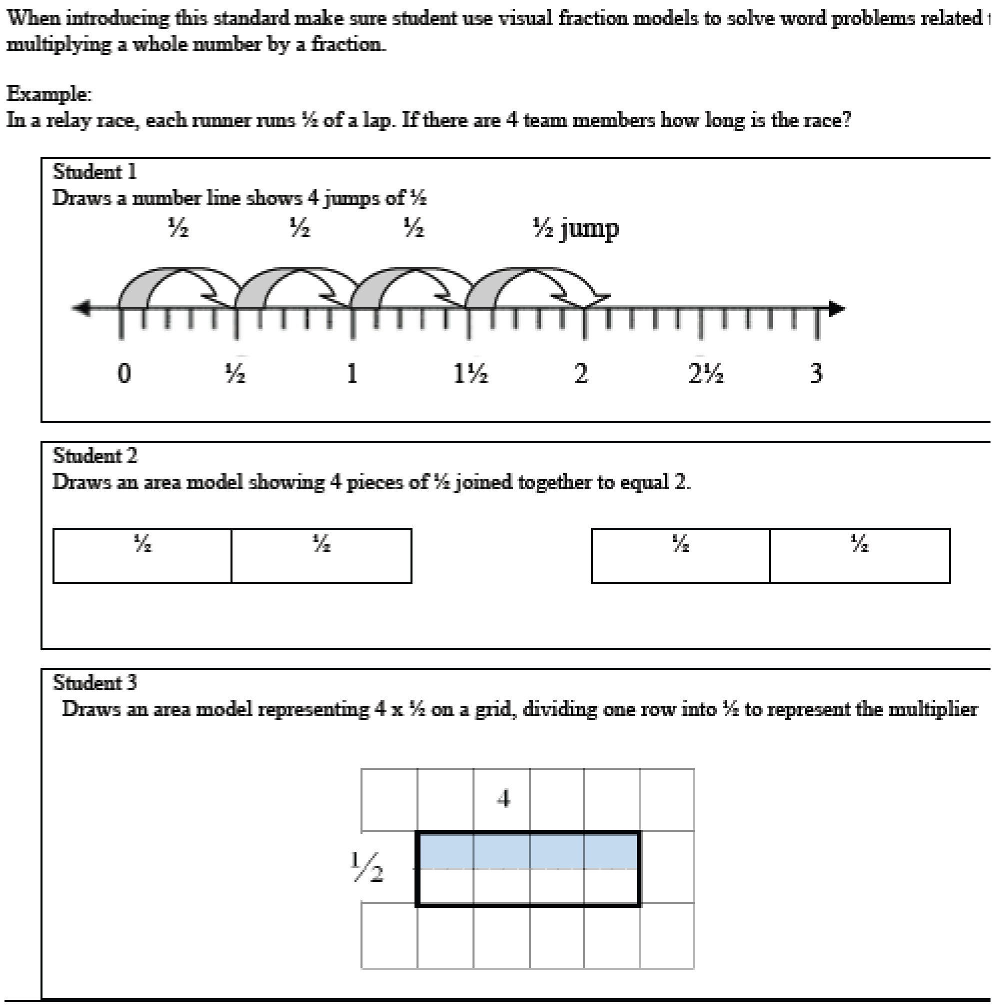
(e.g., by using visual fraction models and equations to represent the problem).	by using properties of operations. 3. Subtract mixed numbers like denominators by using properties of operations. 4. Add mixed numbers with like denominators using relationships between addition and subtraction. 5. Subtract mixed numbers with like denominators using relationships between addition and subtraction. d. 1. Solve word problems involving addition of fractions referring to the same whole with like denominators using visual fraction models. 2. Solve word problems involving addition of fractions referring to the same whole with like denominators using equations to represent the problem. 3. Solve word problems involving subtraction of fractions referring to the same whole with like denominators	Student 2 3 ¾ + 2 = 5 ¾ so 5 ¾ + ¼ = 6 Student 3 3 ¾ = 15/4 and 2 ¼ = 9/4 so 15/4 + 9/4 = 24/4 = 6 Fourth Grade students should be able to decompose and compose fractions with the same denominator. They fractions with the same denominator. Example: $\frac{7}{5}+\frac{4}{5} = \overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{7}+\overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{4} = \frac{\overbrace{1+1+\cdots+1}^{7+4}}{5} = \frac{7+4}{5}.$ (Progressions for the CCSSM: Numbers and Operations Fractions, CCSS Writing Team, May 2011, page 6 and 7) Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract 5/6 from 17/6, they decompose. $\frac{12}{6}+\frac{5}{6}$, so $\frac{17}{6}-\frac{5}{6}=\frac{17-5}{6}=\frac{12}{6}=2.$: Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalent fraction with the same denominator as the fraction. $7\frac{1}{5}=7+\frac{1}{5}=\frac{35}{5}+\frac{1}{5}=\frac{36}{5}.$ Students use this method to add mixed numbers with like denominators. Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition. (Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 6-7) d. The student will solve problems such as: If Joey ate 2/4 of the pizza and Suzie at 1/4 of the pizza, how much pizza did they eat all together? 4/6 + 4/6 + 4/6 =

<table>
<tr><td></td><td>using visual fraction models.
4. Solve word problems involving subtraction of fractions referring to the same whole with like denominators using equations to represent the problem</td><td></td></tr>
<tr><td>4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the</td><td>a. Demonstrate understanding of a multiple of a/b as a multiple of 1/b.
b. Use the understanding of a multiple of a/b as a multiple of 1/b to multiply a fraction by a whole number.
c. 1.Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models to represent the problem.
2. Solve word problems involving multiplication of a fraction by a whole number by using equations to represent the problem. Check to see if the answers make sense.</td><td>This standard builds on students' work of adding fractions and extending that work into multiplicatior
Example:
3/6 = 1/6 + 1/6 + 1/6 = 3 x (1/6)
Number line:


Area model:


Students should see a fraction as the numerator times the unit fraction with the same denominator.
Example:
$\frac{7}{5} = 7 \times \frac{1}{5}$, $\frac{11}{3} = 11 \times \frac{1}{3}$.
(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, pag
This standard extended the idea of multiplication as repeated addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/
6/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number b
fraction.


The same thinking, based on the analogy between fractions and whole numbers, allows students to give mear
to the product of whole number and a fraction.
Example:
$3 \times \frac{2}{5}$ as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$.
.</td></tr>
</table>

problem). Check for the reasonableness of the answer. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?		 When introducing this standard make sure student use visual fraction models to solve word problems related t multiplying a whole number by a fraction. Example: In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race? Student 1 Draws a number line shows 4 jumps of ½ Student 2 Draws an area model showing 4 pieces of ½ joined together to equal 2. Student 3 Draws an area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier
Understand decimal notation for fractions, and compare decimal fractions.		
4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	1. Express a fraction with denominator 10 as an equivalent fraction with denominator 100. 2. Use understanding of fractions with denominator 10 as an equivalent fraction with denominator of 100 to add two fractions with respective denominators of 10 and 100.	3/10 + 4/100 = Express 3/10 as 30/100, and add 3/10 + 4/100 = 30/100 + 4/100= 34/100 75/100 + 8/10= Express 8/10 as 80/100, and add 75/100 + 8/10= 75/100 + 80/100= 155/100= 1 55/100 This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100. Students in fourth grade work with fractions having denominators 10 and 100. Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers.

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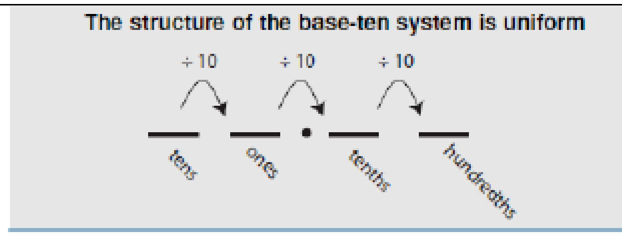
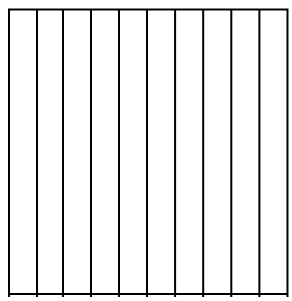
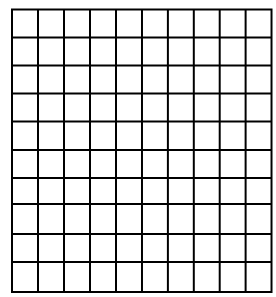
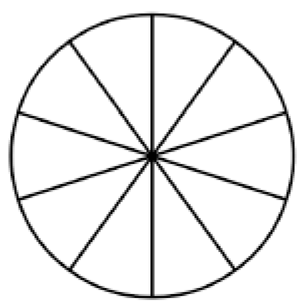
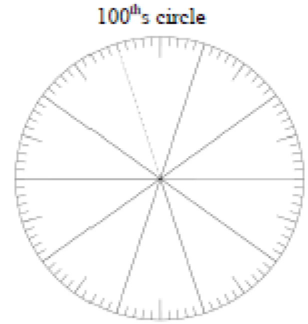


(Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 12)

This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.

<table>
<tr><th>Ones</th><th>.</th><th>Tenths</th><th>Hundredths</th></tr>
</table>
Tenths Grid

.3 = 3 tenths = 3/10
Hundredths Grid

.30 = 30 hundredths = 30/100

Example:
Represent 3 tenths and 30 hundredths on the models below.
10ths circle

100ths circle

</td>
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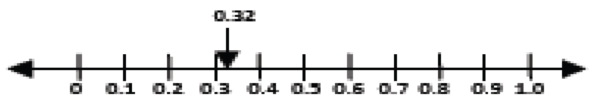
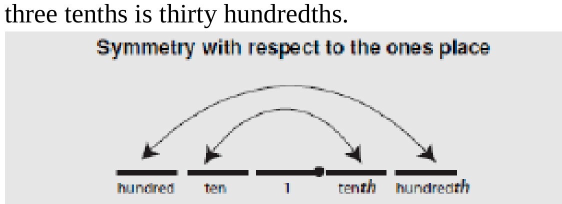
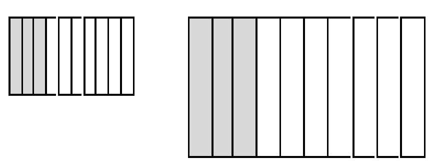
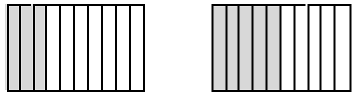
<tr>
<td>4.NF.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</td>
<td>1. Use decimal notation for fractions with denominators 10.
2. Use decimal notation or fractions with denominators 100.</td>
<td>Rewrite 0.62 as 62/100
Describe a length as 0.62 meters
Locate 0.62 on a number line diagram
1/4 = 25/100= .25
Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.
Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths
and rewrite this as 0.32 or represent it on a place value model as shown below.
<table>
<tr><th>Hundreds</th><th>Tens</th><th>Ones</th><th>•</th><th>Tenths</th><th>Hundredths</th></tr>
<tr><td></td><td></td><td></td><td>•</td><td>3</td><td>2</td></tr>
</table>
Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100.
Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be</td>
</tr>
</table>

		placed on the number line near that value. 
4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	1. Compare two decimals to hundredths by reasoning about their size. 2. Prove that comparisons are only valid when the two decimals refer to the same whole. 3. Record comparisons of two decimals using >, =, < symbols. 4. Justify the comparison of two decimals using >, =, < using a visual model.	3.09 Start with the whole number: “Is it closer to 3 or 4?” Go to the tenths: “Is it closer to 3.0 or 3.1?” Go to the hundredths. At each answer, challenge students to defend their choices with the use of a visual model or other conceptual explanation Use a large number line without numerals for students to place numbers for comparison. .56 > .49 .56 = .560 .98< .10 Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks. The decimal point is used to signify the location of the ones place, but its location may suggest there should be a “oneths" place to its right in order to create symmetry with respect to the decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place. Ways of reading decimals aloud vary. Mathematicians and scientists often read 0.15 aloud as “zero point one five" or “point one five." (Decimals smaller than one may be written with or without a zero before the decimal point.) Decimals with many non-zero digits are more easily read aloud in this manner. (For example, the number π, which has infinitely many non-zero digits, begins 3.1415 . . ....) Other ways to read 0.15 aloud are “1 tenth and 5 hundredths” and “15 hundredths,” just as 1,500 is sometimes read “15 hundred” or “1 thousand, 5 hundred.” Similarly, 150 is read “one hundred and fifty” or “a hundred fifty” and understood as 15 tens, as 10 tens and 5 tens, and as 100 + 50. Just as 15 is understood as 15 ones and as 1 ten and 5 ones in computations with whole numbers, 0.15 is viewed as 15 hundredths and as 1 tenth and 5 hundredths in computations with decimals. It takes time to develop understanding and fluency with the different forms. Layered cards for decimals can help students become fluent with decimal equivalencies such as three tenths is thirty hundredths.  (*Progressions for the CCSSM; Number and Operation in Base Ten,* CCSS Writing Team, April 2011, page 12-13) Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.  When the wholes are the same, the decimals or fractions can be compared. Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.



Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and involving time.

4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4-ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).*

1. Know relative sizes of measurement units in the Metric System.
2. Know relative sizes of measurement units in Customary System.
3. Within a single system, compare and record measurements in a larger unit in terms of a smaller unit.
4. Create a two-column table to record measurement equivalents.

Know that 1 ft is 12 times as long as 1 in.
Express the length of a 4 ft snake as 48 in.
Generate a conversion table for feet and inches listing the number pairs (1,12), (2,24), (3,36),...

Super- or subordinate unit	Length in terms of basic unit
kilometer	10^3 or 1000 meters
hectometer	10^2 or 100 meters
decameter	10^1 or 10 meters
meter	1 meter
decimeter	10^{-1} or $\frac{1}{10}$ meters
centimeter	10^{-2} or $\frac{1}{100}$ meters
millimeter	10^{-3} or $\frac{1}{1000}$ meters

Note the similarity to the structure of base-ten units and U.S. currency (see illustrations on p. 12 of the Number and Operations in Base Ten Progression).

Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	
1000	

Foot and inch equivalences

feet	inches
0	0
1	12
2	24
3	

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 20)

Example:
Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	*n* x 3

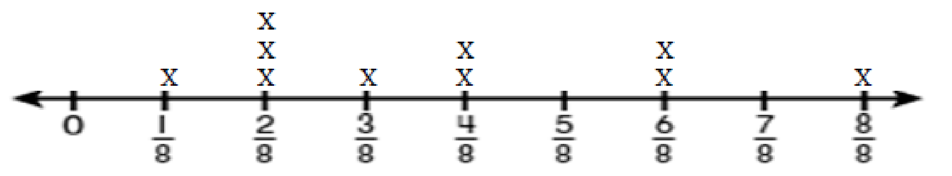
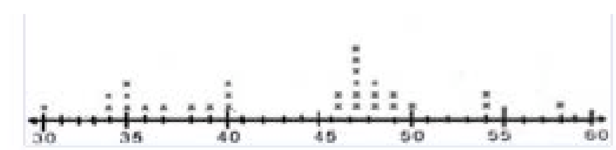
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a

1. Use addition, subtraction, multiplication, and/or division to solve word problems involving distance, including fractions or decimals.
2. Use addition, subtraction, multiplication, and/or division to solve word problems involving intervals of time, including fractions or decimals.
3. Use addition, subtraction,

Given two pieces of 8 1/2 x 11 paper, have students create both a horizontal cylinder and a vertical cylinder.
Explain why or why not both cylinders would how the same amount of liquid.

If Renee has 1/2 of an apple pie and 1/8 of a peach pie and Linda has .50 of an apple pie and wants 1/2 of Renee's piece of peach pie:
A) How much pie do they have altogether?
B) How much of each type of pie do they have?
C) How much pie must Renee give Linda in order for her to half of her slice?

measurement scale.	multiplication, and/or division to solve word problems involving liquid volume, including fractions or decimals. 4. Use addition, subtraction, multiplication, and/or division to solve word problems involving masses of objects, including fractions or decimals. 5. Use addition, subtraction, multiplication, and/or division to solve word problems involving money, including fractions or decimals. 6. Represent measurement quantities using diagrams that feature a measurement scale.	
4.MD.3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	1. Find the area of a rectangle using the area formula for rectangles in real world mathematical problems. 2. Find the perimeter of a rectangle using the perimeter formula for rectangles in real world mathematical problems.	Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. 1. Find the length of a rectangle when: W=5, Area=10 W x L =Area, 5 x L=10, L=2 because 5 x 2 = 10 Give students a loop of string that is exactly 24 units long. Have students decide what different–sized rectangles can be made with a perimeter of 24 inches. Allow a 1 inch grid paper as a tool. Record findings.
4.MD.4. Solve real-world problems involving elapsed time between U.S.	1. Determine start time, elapsed time, and end time to	1. Jan and her family finally had an opportunity to travel to the Lower 48. They left Alaska at 7:30 AM (Alaska Standard Time) and arrived at Atlanta, Georgia, at 10:30 PM (Eastern Standard Time). How many hours did they spend traveling?

time zones (including Alaska Standard time). (L)	the hour, half hour, and nearest minute using a.m. and p.m. 2. Solve real world problems using elapsed time between U.S. time zones (including Alaska Standard time).	__________ hours (***11*** hours) 2. A plane leaves Dallas, Texas, at 2:22 PM (Central Standard Time). It takes the plane 9 hours and 25 minutes to reach Fairbanks, Alaska. At what time does the plane arrive in Fairbanks (Alaska Standard Time)? __________ (***8:47 PM (AST)***
Represent and interpret data.		
4.MD.5. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	1. Create a line plot to display a data set of measurements in fractions of a unit. 2. Solve addition and subtraction fractions problems using information presented in a line plot.	From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. Students measured objects in their desk to the nearest ½, ¼, or 1/8 inch. They displayed their data collected on a line plot. How many object measured ¼ inch? ½ inch? If you put all the objects together end to end what would be the total length of all the objects.  Suppose thirty people live in an apartment building. These are the following ages: 58, 30, 37, 36, 34, 49, 35, 40, 47, 47, 39, 54, 47, 48, 54, 50, 35, 40, 38, 47, 48, 34, 40, 46, 49, 47, 35, 48, 47, 46. Create a line plot to display data. What fractional portion of the total population is 40 or younger? 
4.MD.6. Explain the classification of data from real-world problems shown in graphical representations including the use of terms range and mode with a given set of data. (L)	1. Using a set of data, find the range and mode. 2. Explain the range and mode of a set of data from a real world problem.	Find the range and mode of the number of bear sightings in the Matanuska-Susitna Valley during the summer of 2013. 6, 5, 7, 2, 5, 4, 3
Geometric measurement: understand concepts of angle and measure angles.		
4.MD.7. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement:	1. Understand the definition of an angle. 2. Understand the definition and components of a circle	Students must have practice examining and labeling the components of a circle and recognizing angles formed when rays are drawn from the center of the circle. To make an angle finder: • Cut out two circles using a different color for one of the circles. Fit circles together using the slit. Slide the circle wheel around to form different fractions.

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

(i.e., point of origin, circular arc, interior, exterior).

3. Understand the fractional relationship of angles to circles.

4. Understand the definition of degree as pertaining to a circle.

5. Understand that degrees are one form of angle measurement.

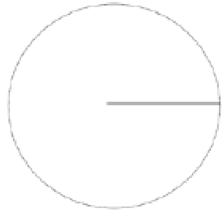
6. Identify the three components of an angle (two rays sharing a common endpoint).

7. Identify a circle as being comprised of 360 one-degree angles.

8. Use models, manipulatives, and pictures to show various types of angles.

9. Use models, manipulatives, and pictures to show degree as the basic unit of measurement of a circle.

10. Use models, manipulatives, and pictures to



This standard brings up a connection between angles and circular measurement (360 degrees).

Angle measure is a “turning point” in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An angle is the union of two rays, a and b, with the same initial point P. The rays can be made to coincide by rotating one to the other about P; this rotation determines the size of the angle between a and b. The rays are sometimes called the sides of the angles. Another way of saying this is that each ray determines a direction and the angle size measures the change from one direction to the other. Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360º

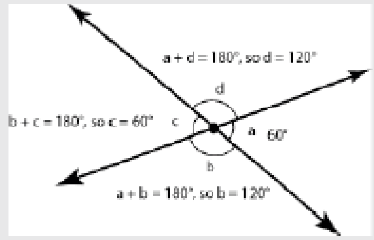
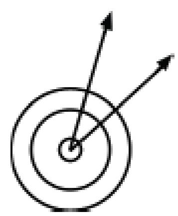
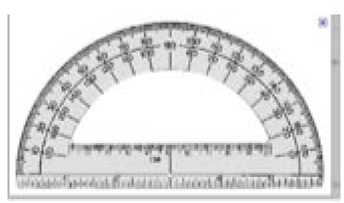
Two angles are called complementary if their measurements have the sum of 90º. Two angles are called supplementary if their measurements have the sum of 180º. Two angles with the same vertex that overlap only at a boundary (i.e., share a side) are called adjacent angles. These terms may come up in classroom discussion, they will not be tested. This concept is developed thoroughly in middle school (7th grade).

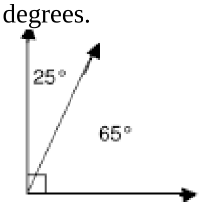
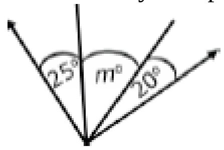
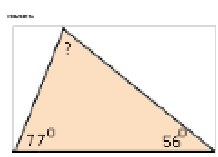
Like length, area, and volume, angle measure is additive: The sum of the measurements of adjacent angles is the measurement of the angle formed by their union. This leads to other important properties. If a right angle is decomposed into two adjacent angles, the sum is 90º, thus they are complementary. Two adjacent angles that compose a “straight angle” of 180º must be supplementary.

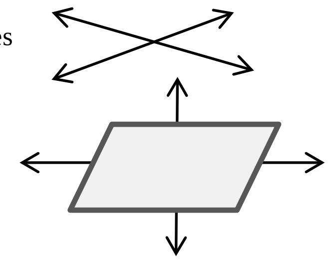
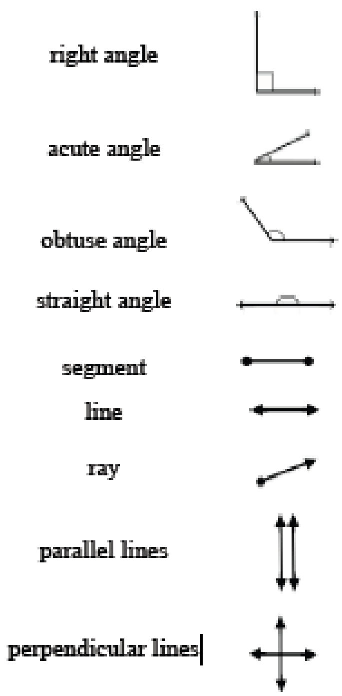
An angle

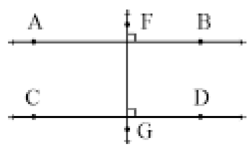
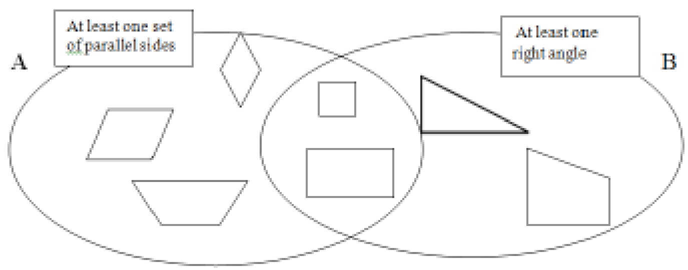
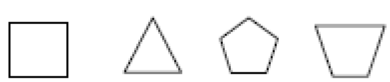
name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

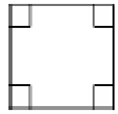
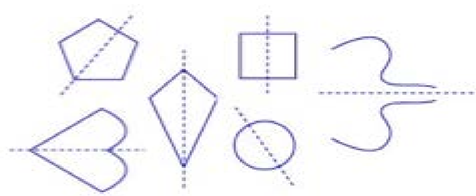
(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23)

	show the relationship between an angle and a circle. b. 1. Understand that degrees are one form of angle measurement. 2. Identify an angle measurement of *n* as being comprised of *n**one-degree angle. 3. Use models, manipulatives, and pictures to show how an angle is measured in *n* degrees.	**Angles created by the intersection of two lines**  When two lines intersect, they form four angles. If the measurement of one is known (e.g., angle *a* is 60°), the measurement of the other three can be determined. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23) **Two representations of three angles**  Initially, some students may correctly compare angle sizes only if all the line segments are the same length (as shown in the top row). If the lengths of the line segments are different (as shown in the bottom row), these students base their judgments on the lengths of the segments, the distances between their endpoints, or even the area of the triangles determined by the drawn arms. They believe that the angles in the bottom row decrease in size from left to right, although they have, respectively, the same angle measurements as those in the top row. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23) The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.  This standard calls for students to explore an angle as a series of "one-degree turns." A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100º, how many one-degree turns has the sprinkler made?
4.MD.8. Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure.	1. Measure angles in whole number degrees using a pro-tractor. 2. Estimate and draw angles of specific measure.	  This angle is _______ degrees.

4.MD.9. Recognize angle measure as additive. When an angle is divided into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).	1. Understand when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. 2. Identify and justify the operation required to find unknown angles from a diagram, real-life problem or a mathematical equation. 3. Use models, manipulatives, diagrams and equations to demonstrate an understanding of additive angle measurement. 4. Using models, manipulatives, and diagrams formulate equations with an unknown value to determine the total measure of the angle.	This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts. A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees.  What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees? If the two rays are perpendicular, what is the value of m?  Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?  A) 40° B) 60° C) 43° D) 47° **Steps to derive** **1** The sum of all the angle measures of a triangle is 180°. **2** $77° + 56° + x° = 180°$ [Equate the sum of angles of the triangle to 180°.] **3** $133° + x° = 180°$ [Add.] **4** $x° = 47°$

Geometry		
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.		
4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in two-dimensional (plane) figures.	1. Draw points and be able to identify points in a two-dimensional figure. 2. Draw rays and be able to identify rays in a two-dimensional figure. 3. Draw angles and be able to identify angles in a two-dimensional figure. 4. Draw perpendicular lines and be able to identify perpendicular lines in a two-dimensional figure. 5. Draw parallel lines and be able to identify parallel lines in a two-dimensional figure. 6. Draw lines and line segments, and be able to identify in two-dimensional figures.	Point ● Intersecting lines Plane  

4.G.2. Classify two-dimensional (plane) figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	1. Classify two-dimensional figures based on presence or absence of parallel lines. 2. Classify two-dimensional figures based on presence or absence of perpendicular lines. 3. Classify two-dimensional figures based on presence or absence of angles of a specific size. 4. Identify and classify right triangles in their own category.	Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement. Parallel or Perpendicular Lines: Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles. A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other. Parallel and perpendicular lines are shown below:  This standard calls for students to sort objects based on parallelism, perpendicularity and angle types. Which figure in the Venn diagram below is in the wrong place, explain how do you know?  Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles. Draw and name a figure that has two parallel sides and exactly 2 right angles. For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example. • A parallelogram with exactly one right angle. • An isosceles right triangle. • A rectangle that is not a parallelogram. (impossible) • Every square is a quadrilateral. • Every trapezoid is a parallelogram. Identify which of these shapes have perpendicular or parallel sides and justify your selection.  A possible justification that students might give is: The square has perpendicular lines because the sides meet at a corner, forming right angles.

		 Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.
4.G.3. Recognize a line of symmetry for a two-dimensional (plane) figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	1. Recognize and create a line of symmetry as a line across the figure by folding figure into matching parts. 2. Identify symmetric figures. 3. Draw symmetric figures.	 For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions. Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.