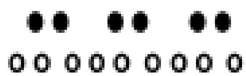
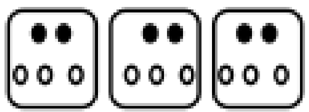
6th Grade

Ratios and Proportional Relationships 6.RP

Ratios and Proportional Relationships.

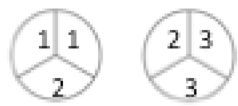
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-to- whole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

State of Alaska Standard	What does this standard mean that a student will know and be able to do?
6.RP.1. Write and describe the relationship in real life context between two quantities using ratio language. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**6.RP.1** A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). <u>Example 1:</u> A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: 6, 6 to 9 *or* 6:9. If 9 the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as  These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, 2, 2 to 3 or 2:3.  Students should be able to identify and describe any ratio using "For every _____, there are _____" In the example above, the ratio could be expressed saying, "For every 2 goldfish, there are 3 guppies".
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per*	**6.RP.2** A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time. Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

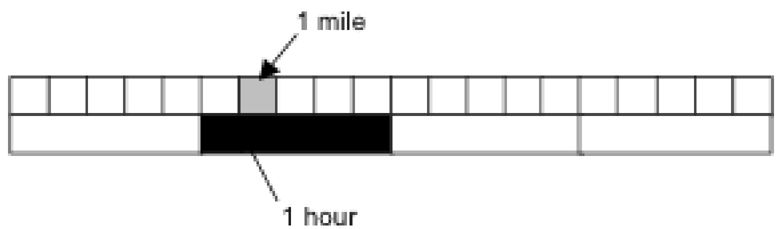
<table>
<tr>
<td>hamburger."1

1 Expectations for unit rates in this grade are limited to non-complex fractions.</td>
<td>Example 1: There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate)
Solution: This can be modeled as shown below to show that there is 2/3 of a cookie for 1 student, so the unit rate is 2/3: 1.



Example 2: On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\,hr}{1\,mi}$. Students can represent the relationship between 20 miles and 4 hours.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<tr>
<td>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare</td>
<td>6.RP.3 Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is not expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard.

Example 1: At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54.

Solution: To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various</td>
</tr>
</table>

ratios.	contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. 1 • 7 = 7; 6 • 7 = 42). Red numbers indicate solutions.

Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

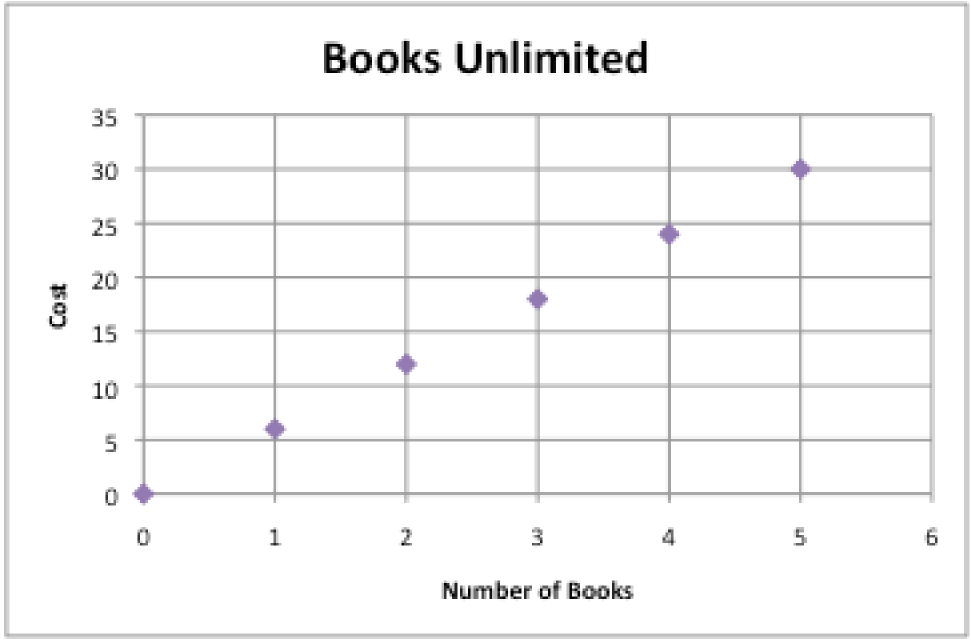
Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7^{th} grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.

The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



<u>Example 2:</u>
Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.
The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

<u>Example 3:</u>
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:
There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

<u>Example 4:</u>
Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:
There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate how many lawns*

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

<u>Example 1:</u> In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies

<table>
<tr>
<td>could be mowed in 35 hours? At what rate were lawns being mowed?</td>
<td>would be needed for 9 cups of peanuts?
<table>
<tr><th>Peanuts</th><th>Chocolate</th></tr>
<tr><td></td><td></td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td></td><td></td></tr>
</table>
Solution:
One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine ($9 \cdot \frac{2}{3}$), giving 6 cups of chocolate.

<u>Example 2:</u>
If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:
The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.</td>
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<td>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding whole, given and part percent.</td>
<td>This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.

Students use ratios to identify percents.
Example 1: What percent is 12 out of 25? Solution: One possible solution method is to set up a ratio table: Multiply 25 by 4 to get100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%
<table>
<tr><th>Part</th><th>Whole</th></tr>
<tr><td></td><td></td></tr>
<tr><td>12</td><td>25</td></tr>
<tr><td>?</td><td>100</td></tr>
</table>
Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

<u>Example 2:</u>
What is 40% of 30?
Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.
See the web link below for more information.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Students also determine the whole amount, given a part and the percent.

<u>Example 3:</u>
If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class</td>
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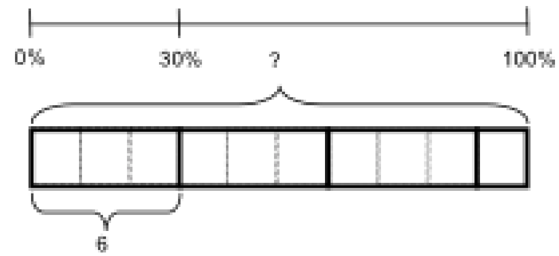
</table>

if 6 like chocolate ice cream?



(*Solution:* 20)

<u>Example 4:</u>
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?

Solution:

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

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d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12\text{ inches}}{1\text{ foot}}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1\text{ foot}}{12\text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel".

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

<u>Example 1:</u>
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm.

Solution:

<u>Example 1:</u>
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm.
Solution:

$$7\text{ feet} \times \frac{12\text{ inches}}{1\text{ foot}} \times \frac{2.54\text{ cm}}{1\text{ inch}} = 7\,\cancel{\text{feet}} \times \frac{12\,\cancel{\text{inches}}}{1\,\cancel{\text{foot}}} \times \frac{2.54\text{ cm}}{1\,\cancel{\text{inch}}} = 7 \times 12 \times 2.54\text{ cm} = 213.36\text{ cm}$$

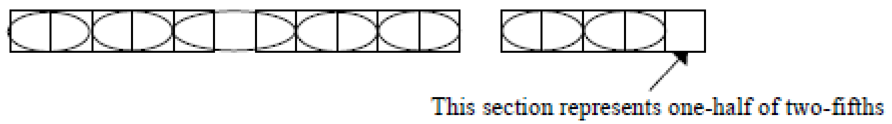
Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

<table>
<tr><td colspan="2">Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: rational numbers, integers, additive inverse</td></tr>
<tr><td>State of Alaska Standard</td><td>What does this standard mean that a student will know and be able to do?</td></tr>
<tr><td>6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem). For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?</td><td>6.NS.1 In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division.

<u>Example 1:</u> Students understand that a division problem such as 3 ÷ 2/5 is asking, "how many 2/5 are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of 1/2. Therefore, 3 ÷2/5= 7 1/2, meaning there are 7 1/2 groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



Students also write contextual problems for fraction division problems. For example, the problem, 2/3 ÷ 1/6 can be illustrated with the following word problems:</td></tr>
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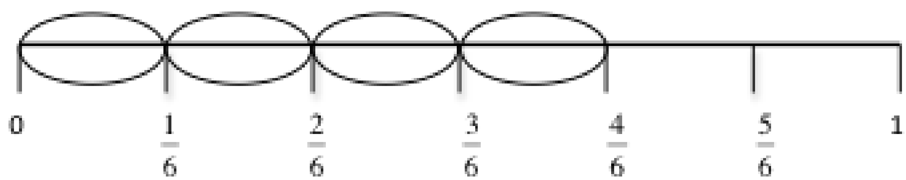
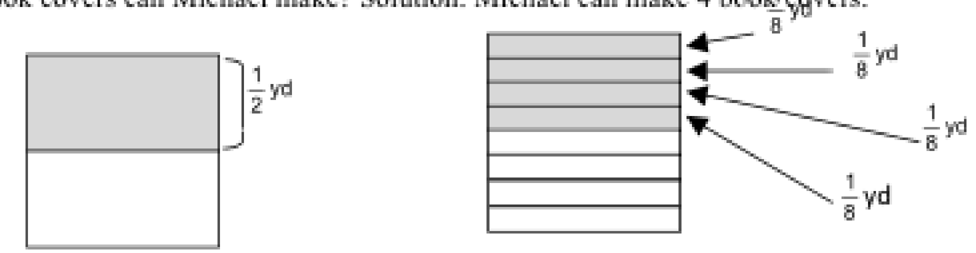
	Example 2: Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make? This problem can be modeled using a number line. a. Start with a number line divided into thirds.  b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.  c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards. Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.



Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

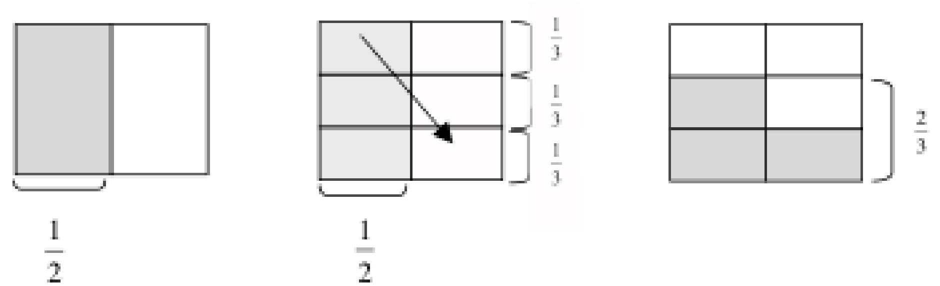
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.2. Fluently multiply and divide multi-	**6.NS.2** In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value.

<table>
<tr>
<td>digit whole numbers using the standard algorithm. Express the remainder as a whole number, decimal, or simplified fraction; explain or justify your choice based on the context of the problem.</td>
<td><u>Example 1:</u>
When dividing 32 into 8456, students should say, “there are 200 thirty-twos in 8456” as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64.
<table>
<tr><td>2
32)8456</td><td>There are 200 thirty twos in 8456.</td></tr>
<tr><td>2
32)8456
6400
2056</td><td>200 times 32 is 6400.
8456 minus 6400 is 2056.</td></tr>
<tr><td>26
32)8456
6400
2056</td><td>There are 60 thirty twos in 2056.</td></tr>
<tr><td>26
32)8456
6400
2056
1920
136</td><td>60 times 32 is 1920.
2056 minus 1920 is 136.</td></tr>
<tr><td>264
32)8456
6400
2056
1920
136
128</td><td>There are 4 thirty twos in 136.
4 times 32 is 128.</td></tr>
<tr><td>264
32)8456
6400
2056
1920
136
128
8</td><td>The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8.

This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8.

8456 = 264 * 32 + 8</td></tr>
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<tr>
<td>6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. Express the remainder as a terminating decimal, or a repeating decimal, or rounded to a designated place value.</td>
<td>6.NS.3 Procedural fluency is defined by the Common Core as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”. In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations.
The use of estimation strategies supports student understanding of decimal operations.

<u>Example 1:</u>
First estimate the sum of 12.3 and 9.75.
Solution: An estimate of the sum would be 12 + 10 or 22. Student could also state if their estimate is high or low.

Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.</td>
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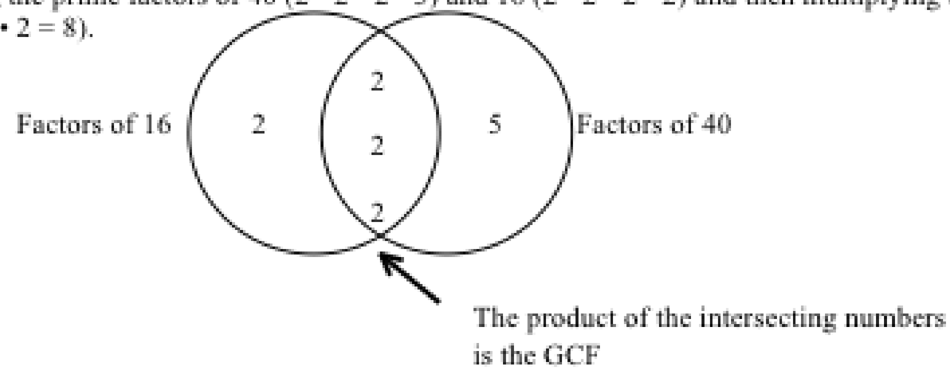
<td>6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).</td>
<td>
In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students will find the greatest common factor of two whole numbers less than or equal to 100.
For example, the greatest common factor of 40 and 16 can be found by

listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.
listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).



Students also understand that the greatest common factor of two prime numbers is 1.
<u>Example 1</u>:
What is the greatest common factor (GCF) of 18 and 24?
Solution: $2 \cdot 3^2 = 18$ and $2^3 \cdot 3 = 24$. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making $2 \cdot 3$ or 6 the GCF.
Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.
Example 2: Use the greatest common factor and the distributive property to find the sum of 36 and 8.
36 + 8 = 4 (9) + 4(2)
44 = 4 (9 + 2)
44 = 4 (11)
44 = 44 ☐
</td>
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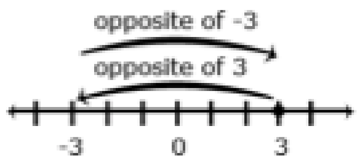
	<u>Example 3:</u> Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used. a. What is the greatest number of students that can attend the picnic? b. How many bags of chips will each student receive? c. How many hotdogs will each student receive? *Solution:* a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF). b. Each student would receive 4 bags of chips. c. Each student would receive 5 hot dogs. Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by 1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or 2) using the prime factorization. Step 1: find the prime factors of 6 and 8. 6 = 2 • 3 8 = 2 • 2 • 2 Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2 Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors. <u>Example 4:</u> The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again? *Solution:* The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20. One way to find the least common multiple is to find the prime factorization of each number: $2^2 \cdot 5 = 20$ and $3 \cdot 5 = 15$. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 (2 • 2 • 5). To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5 (2 • 2 • 3 • 5) or 60.

Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS.5 Understand that positive and negative numbers describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explain the meaning of 0 in each situation.	**6.NS.5** Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. <u>Example 1:</u> a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; Recognize that the opposite of the opposite of a number is the number itself [e.g., –(–3) = 3] and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	**6.NS.6** In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In the example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4  **<u>Example 1:</u>** **What is the opposite of 2 ½? Explain your answer/** ***Solution:* -2½ because it is the same distance from 0 on the opposite side.** x-axis and y-axis should now include negatives, and be introduced to the Cartesian Coordinate system. Students recognize the point where the x-axis and the y-axis intersec-½t as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (- , +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pair (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change in the x-coordinates from (-2 ,4) and (2, 4) represents a reflection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2,4)], the order pair has been reflected across both axes. <u>Example 1:</u> Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the x-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original points and the reflected point?

	(½, -3½) (-½, -3) (0.25, 0.75) *Solution*: The coordinates of the reflected points would be (½, 3½) (-½, 3) (0.25, 0.75). Note that the y-coordinates are opposites. <u>Example 2:</u> Students place the following numbers on a number line; -4.5, 2, 3.2, -3 $^{3}/_{5}$, 0.2, -2, $^{11}/_{2}$. Based on number line placement, numbers can be placed in order. *Solution:* The numbers in order from least to greatest are: -4.5, -3 $^{3}/_{5}$, -2, 0.2, 2, 3.2, $^{11}/_{2}$ Students place each of these numbers on a number line to justify this order.
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars*	6 NS 7 a. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left of the number line. Common Models to represent and compare integers include number line models, temperature models, and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of the arrow. The number line can also be viewed as a thermometer where each point of the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers**. Operations with integers are not the expectations at this level.** In working with number line models, students internalized the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same line, they are able to write inequalities and make statements about the relationship between two numbers.

Case 1: Two positive numbers

5 > 3
5 is greater than 3
3 is less than 5

Case 2: One positive and one negative number

3 > -3
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers

-3 > -5
negative 3 is greater than negative 5
negative 5 is less than negative 3

__Example 1:__
Write a statement to compare – 4 ½ and –2. Explain your answer.

Solution:
– 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line

Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

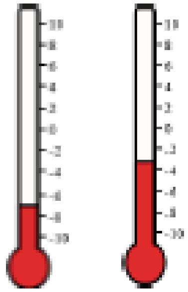
b.

Students write statements using $<$ or $>$ to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than".

Example 1:
The balance in Sue's checkbook was –\$12.55. The balance in John's checkbook was –\$10.45. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: $-12.55 < -10.45$, Sue owes more than John. The interpretation could also be "John owes less than Sue".

Example 2:
One of the thermometers shows -3°C and the other shows -7°C.
Which thermometer shows which temperature?
Which is the colder temperature? How much colder?
Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:

- The thermometer on the left is -7; right is -3
- The left thermometer is colder by 4 degrees
- Either $-7 < -3$ or $-3 > -7$

Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3:
A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature:

Albany	5°
Anchorage	-6°
Buffalo	-7°
Juneau	-9°
Reno	12°

Solution:

Juneau	-9°
Buffalo	-7°
Anchorage	-6°
Albany	5°
Reno	12°

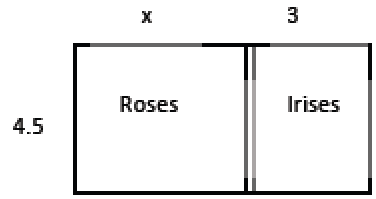
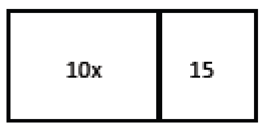
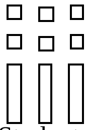
c.

	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. <u>Example 1:</u> Which numbers have an absolute value of 7 *Solution:* 7 and −7 since both numbers have a distance of 7 units from 0 on the number line. <u>Example 2:</u> What is the $\|-3\frac{1}{2}\|$? *Solution:* $3\frac{1}{2}$ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write |−900| = 900 to describe the distance below sea level. d. When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left of the number line), the value of the number decreases. For example, -24 is less than -14 because -24 is located to the left of -14 on the number line. In terms of absolute value (or distance) the absolute value of -24 is greater than the absolute value of -14. For negative numbers, as the absolute value increases, the value of the negative number decreases.
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	6 NS 8: Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). <u>Example 1:</u> What is the distance between (-5,2) and (-9,2)? *Solution:* The distance would be 4 units. This would be a horizontal line since the y=coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between -5 and -9. Students could also recognize that -5 is 5 units from 0 and that -9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances9 and 5. (|9|-|5|). Coordinates could also be in two quadrants and include rational numbers. **<u>Example 2:</u>** What is the distance between (3,-5½) and (3, 2¼)? *Solution:* The distance between (3,-5½) and (3, 2¼) would be 7¾ units. This would be a vertical line since the x-coordinates are the same. The distance can be found by using a number line to count from -5½ to 2¼ or by recognizing that the absolute value from -5½ to 0 is 5½ units and the distance from 0 to 2¼ is 2¼ units so the total distance would be 5½ + 2¼ or 7¾ units. Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.
Expressions and Equations 6.EE	
Apply and extend previous understandings of arithmetic to algebraic expressions.	
Mathematically proficient students communicate precisely by engaging in discussions about their reasoning using appropriate mathematical language. The terms students should	

learn to use with increasing precision with this cluster are: exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables	
State of Alaska Standard	What does this standard mean that a student will know and be able to do?
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. *For example, multiply by powers of 10 and products of numbers using exponents.* ($7^3 = 7•7•7$)	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or positive fraction (i.e. $½^5$ can be written ½•½•½•½•½ which has the same value as $^1/_{32}$.). Students recognize that an expression with a variable represents the same mathematics (i.e. x^5 can be written as x• x• x• x•x) and write algebraic expressions from verbal expressions. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { } and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> What is the value: • $0,2^3$ *Solution:* 0.008 • $5 + 2^4 • 6$ *Solution:* 101 • $7^2 – 24 ÷ 3 + 26$ *Solution:* 67 <u>Example 2:</u> What is the area of a square with a side length of 3x? *Solution:* $3x • 3x = 9x^2$ <u>Example 3:</u> $4^x = 64$
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* *2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions and formulas. Include formulas used in real-world problems. Perform arithmetic operations,	Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "6 times any number, n" could be represented with 6n and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. <u>Example Set 1</u>: Students read algebraic expressions: • r + 13 as "some number plus 13" or as "r plus 13" • 7 • p as " 7 times some number" or as "7 times p" • $\frac{s}{6}$ and s÷6 as "some number divided by 6" or as " s divided by 6" <u>Example Set 2</u>: Students write algebraic expressions: • 7 less than 3 times a number *Solution*: 3x-7 • 3 times the sum of a number and 5

including those involving whole number exponents, in the conventional order with or without parentheses. (Order of Operations)	*Solution:*3(x+5) • 7 less than the product of 2 and a number *Solution*: 2x-7 • Twice the difference between a number and 5 *Solution*: 2(z-5) • The quotient of the sum of x plus 4 and 2 *Solution*: $\frac{x+4}{2}$ Students can describe expressions such as 3(2+6) as the product of two factors: 3 and (2+6). The quantity (2+6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product , and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent. Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y There are 4 terms, x^2, 5y, 3x, and 6 There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represents 5y's or 5•y. There is one constant term, 6. The expression represents a sum of all 4 terms. c. Students evaluate algebraic expressions, using order of operations as needed. A problem such as example 1 below requires students to demonstrate that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. Order of Operations is introduced throughout elementary grades, including the use of grouping symbols ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. Example 1: Evaluate the expression 3x + 2y when x is equal to 4 and y is equal to 2.4 *Solution:* 3 • 4 + 2 · 2.4 12 + 4.8 16.8 Example 2: Evaluate 5(n + 3) – 7n, when n=½ *Solution*: 5(½ + 3) – 7(½)

	5(3½) - 3½ Note: 7(½) = $\frac{7}{2}$ = 3½ 17½ - 3½ Students may also reason that 5 groups of 3½ take away 1 group of 3½ 14 would give 4 groups of 3½. Multiply 4 times 3½ to get 14. <u>Example 3:</u> Evaluate 7xy when x=2.5 and y=9 *Solution:* Students recognize that two or more terms written together indicates multiplication. 7 (2.5) (9) 157.5 In 5th grade students worked with grouping symbols ( ), [ ], { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol. <u>Example 4:</u> Evaluate the following expression when x=4 and y=2 $\frac{X^2 + y^3}{3}$ $\frac{4^2 + 2^3}{3}$ Substitute the values for x and y $\frac{16 + 8}{3}$ evaluate the powers $\frac{24}{3}$ divide 24 by 3 8 Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. <u>Example 5</u>: It costs $100 to rent the skating rink plus $5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25 people? *Solution*: The cost for any number (n) of people could be found by the expression, 100 + 5n. To find the cost of 25 people substitute 25 in for n and solve to get 100 + 5 ·25 = 225. <u>Example 6</u>: The expression c + 0,07c can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25. *Solution*: c + 0.07c 25 + 0.07(25) 25 + 1.75 26.75

6.EE.3 Apply the properties of operations to generate equivalent expressions. Model (e.g., manipulatives, graph paper) and apply the distributive, commutative, identity, and inverse properties with integers and variables by writing equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x.*	**Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary students illustrate the distributive property with variables.** **Properties are introduced throughout elementary grades (3.OA.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4).** **<u>Example 1:</u>** **Given that the width is 4.5 units and the length can be represented by x + 2, the area of the flowers below can be expressed as 4.5(x + 3) or 4.5x + 13.5**  When given an expression representing area, students need to find the factors. <u>Example2:</u> The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).  <u>Example 3:</u> Students use their understanding of multiplication to interpret 3 (2 + x) as 3 groups of (2 + x). They use the model to represent x, and make an array to show the meaning of 3(2 +x). They can explain why it makes sense that 3(2 + x) is equal to 6 + 3x. An array with 3 columns and x + 2 in each column:  Students interpret y as referring to one y. They can reason that one y plus one y plus one y must be 3y. They also use the distributive property, the multiplication identity property of 1, and the commutative property for multiplication to prove that y + y + y = 3y. <u>Example 4</u>: Prove that y + y + y = 3y

Solution:
y + y + y
y • 1+ y • 1+ y • 1 Multiplicative Identity
y· (1 + 1 + 1) Distributive
y·3
3y Commutative Property

6.EE.4
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x; however $3x + 4x^2$ are not like terms since the exponents with the x are not the same.

This concept can be illustrated by substituting in a value for x. For example, 9x – 3x = 6x not 6. Choosing a value for x, such as 2, can prove non-equivalence.

9(2) -3(2) =6(2)	however	$9(2) – 3(2) \neq 6$
18 – 6 = 12		$18 – 6 \neq 6$
12=12		$12 \neq 6$

Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

<u>Example 1:</u>
Are the expressions equivalent? Explain your answer?
4m + 8   4(m + 2)   3m + 8 + m   2 + 2m + m + 6 + m

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2m +m +m +2 + 6* *4m + 8*	*Combined like terms* *Combined like terms*

Reason about and solve one-variable equations and inequalities.

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
For example: does 5 make 3x > 7 true?

In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.

<u>**Example 1:**</u>
Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation 26 + n = 100 where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100." Students ask themselves "what number was

	added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find the solution to the problem. • Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74. • Use knowledge of fact families to write related equations: N + 26 = 100, 100 – n = 26, 100 – 26 = n. Select the equation that helps find n easily. • Use knowledge of inverse operations: since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n • Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balanced. • Bar model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100. [100] [26 | *n*] *Solutions:* Students recognize the value of 74 would make a true statement if substituted for the variable. 26 + n = 100 26 + 74 = 100 100 = 100 ✓ <u>Example 2:</u> The equation 0.44 s =11 where s represents the number of stamps in a booklet. The booklet of stamps cost 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution. *Solution:* There are 25 stamps in a booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11. By substituting 25 in for s and then multiplying, I get 11. 0.44 (25) = 11 11=11✓ <u>Example 3:</u> Twelve is less than 3 times another number can be shown by the inequality 12 < 3n. What numbers could possibly make this a true statement? *Solution:* Since 3 · 4 is equal to 12 I know the value must be greater than 4.Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, 5¾, and 200. Given a set of values, students identify the values that make the inequality true.
6.EE.6 Use variables to represent numbers and write expressions when solving a real-	Students write expressions to represent various real-world situations. <u>Example Set 1</u>:

world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	- Write an expression to represent Susan's age in three years, when a represents her present age. - Write an expression to represent the number of wheels, w, on any number of bicycles. - Write an expression to represent the value of any number of quarters, q. *Solutions*: - a + 3 2n 0.25q Given a contextual situation, students define variables and write an expression to represent the situation. <u>Example 2</u>: The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people. N = the number of people 100 + 5n Students are not expected to solve, Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has $1/3$ the amount of Sally. If s represents the number of bracelets Sally has, the $1/3$ s, or $s/3$ represents the amount Jane has. Connecting writing expressions with story problems and / or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. <u>Example Set 3:</u> - Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. *Solution*: 2c + 3 where c represents the number of crayons that Elizabeth has. - An amusement park charges $28 to enter and $0.35 per ticket. Write and algebraic expression to represent the total amount spent. *Solution:*28 + 0.35t where t represents the number of tickets purchased - Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned. *Solution:* 15h + 20 = 85 where h is the number of hours worked.
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which *p*, *q* and *x* are all nonnegative rational numbers.	Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, x + 4, any value can be substituted for the x to generate a numerical answer; however, in the equation x + 4 = 6, there is only one value that can be used to get a 6. Problems should be in context when possible and used only one variable. Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions.

Students recognize that dividing by 6 and multiplying by $^{1}/_{6}$ produces the same result. For example, $^{x}/_{6}$ = 9 and $\frac{1}{6}$ x = 9 will produce the same result.

Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem.

<u>Example 1</u>:
Meagan spent $56.58 on three pairs of jeans. If each pair of jeans cost the same amount, write an algebraic equations that represents this situation and solve to determine how much one pair of jeans cost.

$56.58		
J	J	J

Sample Solution:
Students might say, “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans cost the same amount of money. The bar model represents the equation 3j=$56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pair of jeans. I know that it will be more than $10 each because 10 · 3 is only 30 but less than $20 each because 20 • 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. Thant is $9 more dollars. I only have $2.58 left. I continue until the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15 + 3 + 0.86). I double check that the jeans cost $18.86 each because $18.86 ·3 is $56.58.”

<u>Example 2:</u>
Julie gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.9	6.50	money left over (m)

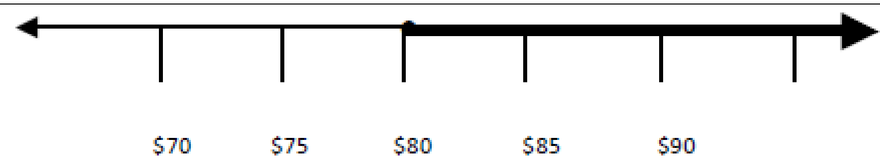
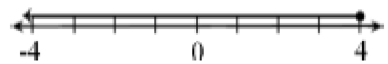
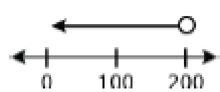
Solution:
20 = 1.99 + 6.50 + x, x = $11.51

6.EE.8
Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Many real-world situations are represented by inequalities. Student write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contecxtual and mathematical situations.

<u>Example 1:</u>
The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, m, the class still needs to raise. Represent this inequality on a number line.

Solution:
The inequality m≥ $80 represents the situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

	 A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions. <u>Example 2:</u> Graph $x \leq 4$ *Solution:*  <u>Example 3:</u> The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line. *Solution:* 200 > x, where x is the amount spent on groceries. 
Represent and analyze quantitative relationships between dependent and independent variables.	
6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how the y variable changes?) Relationships should be proportional with the line passing through the origin. Students, should also be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and / or table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing in words, a table, an equation or a graph. Translating between multiple representations help students understand that each form represents the same relationship and provides a different perspective.

<u>Example 1:</u>
What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

Solution:
Y = 2.5x

Geometry 6.G

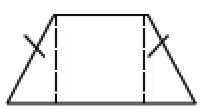
Solve real-life and mathematical problems involving area, surface are and volume.

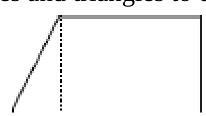
Mathematically proficient students communicate precisely by engaging in discussions about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, rhombi, right rectangular prism.

State of Alaska Standard	What students should know and be able to do
6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing or decomposing into other polygons (e.g., rectangles and triangles). Apply these techniques in the context of solving real-world and mathematical problems.	Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area) and the figure. This understanding should be all students.

Finding the area of triangles is introduced in relationship to the area of rectangles-a rectangle can be decomposed into two congruent triangles, deducing that the area of the triangle is ½ of the area of the rectangle. The area of a rectangle can be found by multiplying base and height; therefore , the area of a triangle is ½ bh.

Students decompose shapes into rectangles and triangles to determine the area. See examples below.


Isosceles trapezoid


Right trapezoid

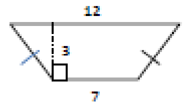
Note: Students recognize that marks on the isosceles trapezoid indicating the two sides have equal measure.

<u>Example 1:</u>
Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

Solution:
Students should understand that the hypotenuse is the longest side of a right triangle. The base and height would form a 90° angle and would be used to find the area using the formula for the area of a triangle.
A= ½ (3units)(4units)
A = ½ (12 units^2)
A = 6 units^2

<u>Example 2:</u>

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:
The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2.

The triangle on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both sides of the triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5units)(3 units) which equals 3.75 units^2. Therefore, the trapezoid would be 21 units^2, plus 3.75 units^2 plus 3.75 units^2 for a total of 28.5 units^2

Example 3:
A rectangle measures 3 inches by 4 inches. If the length of each side double, what is the effect on the area?

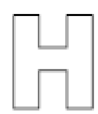
Solution:
The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches$^{2.}$ The area of the new rectangle is 48 inches^2 , therefore, the area increased 4 times (quadrupled). Students can also represent this by creating a drawing.

Example 4:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution:
Change the dimensions of the bulletin board to inches. The area of the board would be 48 inches by 36 inches or 1728 inches^2 The area of one index card 12 inches^2. Divide 1728 inches^2 by 24 inches^2 to get 72 index cards.

Example 5:
The sixth grade class at Houston Middle is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

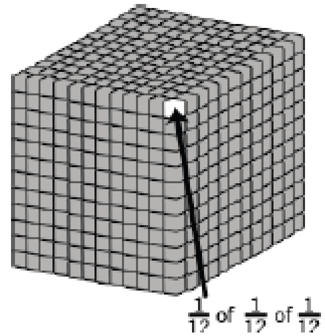
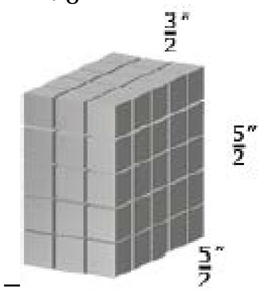
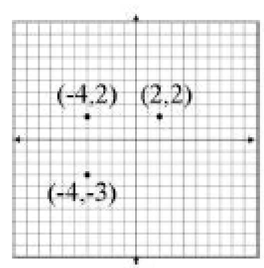
1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?



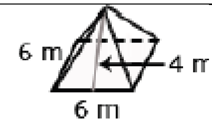
Solution:

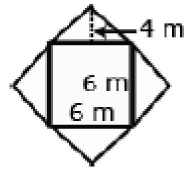
1. One solution would be to decompose the H into two tall rectangles measuring 10 feet by 25 feet and one small rectangle measuring 2.5 feet by 5 feet. The area of the tall rectangles would be 25 feet^2 each and the smaller rectangle would be 12.5 feet^2. So the area of the H would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5 ft^2.
2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5 ft by 5ft. Cut 2 pieces of wood in half to

	create four pieces of 5 ft by 2.5 ft. These pieces will make the 2 taller rectangles. A third piece would be cut to measure 5 ft by 2.5 ft to create the center piece. <u>Example 6:</u> A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border? *Solution:* Two sides 4 ft by 2 ft would be 8 ft^2 by 2 or 16 ft^2 Two sides 3 ft by 2 ft would be 6 ft^2 by 2 or 12 ft^2 Four corners measuring 2 ft by 2 ft would be 4 ft^2 by 4 or 16 ft^2 The total area of the boarder would be 16 ft^2 + 12 ft^2 + 16 ft^2 or 44 ft^2
6.G.2 Apply the standard formulas to find volumes of prisms. Use attributes and properties (including shapes of bases) of prisms to identify compare or describe three-dimensional figures including prisms and cylinders.	Students have already calculated the volume or right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V=Bh. The unit cube was 1 · 1•1. In 6^{th} grade the unit cube will have fractional edge lengths.(i.e. ½ ·½ ·½) Students find the volume of the right rectangular prism with these unit cubes. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, student *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling the box with unit cubes and the volume formula. In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes. <u>Example 1:</u> A right rectangular prism has edges of 1 ¼", 1" and 1 ½". How many cubes with side lengths of ¼ would be needed to fill the prism? What is the volume of the prism? *Solution:* The number of ¼ " cubes can be found by recognizing the smaller cubes would be ¼ " on all edges, changing the dimensions to $^5/_4$", $^4/_4$" and $^6/_4$". The number of one-fourth inch unit cubes making up the prism is 120(5•4·6). Each smaller cube has a volume of $^1/_{64}$ ($^1/_4$•$^1/_4$•$^1/_4$), meaning 64 small cubes would make up the unit cube. Therefore, the volume is $^5/_4$· $^4/_4$•$^6/_4$ or $^{120}/_{64}$ (120 smaller cubes with volumes of $^1/_{64}$ or $1^{56}/_{64}$ →1 unit cube with 56 smaller cubes with a volume of $^1/_{64}$. <u>Example 2:</u> The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $^1/_{12}ft^3$.

	 Example 3: The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is ½ in on each side. Students work with the model to illustrate $\frac{3}{2} \bullet \frac{5}{2} \cdot \frac{5}{2} = (3 \bullet 5 \cdot 5) \bullet \frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8} in^3$ because 8 of them fit in a unit cube. 
6.G.3 Draw polygons in the coordinate planes given coordinates for the vertices; determine the length of a side joining the coordinates of vertices with the same first or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, -1) and (2, 4), then the students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, so 5.If both the y-coordinates are the same (-5, 4) and (2,4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, so 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane. Example 1: If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle. 

	Solution: To determine the distance along the x-axis between point(-4, 2) and (2,2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of 0, so the two points are a total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2|. The length is 6 and the width is 5. The fourth vertex would be (2, -3) The area would be 5·6 or 30 units^2 The perimeter would be 2l +2w, 2(5) + 2(6) or 22 units. <u>Example 2:</u> On a map, the library is located at (-2,2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile. 1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know? 2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park? *Solution:* 1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same y-coordinate. The distance between the x-coordinates is 2 (from -2 to) 2. The three locations form a right triangle. The area is 2 mi^2.
6.G. 4 Represent three-dimensional figures (e.g. prisms) using nets made up of rectangles and triangles, and use the nets to find surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	A net is a two- dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and / or triangle and add these sums together to find the surface area of the figure. Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices and faces. Solid include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area Students also describe the types of faces needed to create a three=dimensional figure. Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create specific three-dimensional figure. <u>Example 1:</u> Describe the shape of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? <u>Example 2:</u> Create the net for a given prism or pyramid, and then use the net to calculate the surface area.





6.G.5 Identify, compare or describe attributes and properties of circles (radius and diameter)	

Statistic and Probability 6.SP

Solve real-life and mathematical problems involving area, surface are and volume.

Mathematically proficient students communicate precisely by engaging in discussions about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision are: statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean.

State of Alaska Standard	What does this standard mean that a student will know and be able to do?
6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the questions and accounts for it in the answer. For example, “ How am I ?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in student’s ages.	Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from on individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources i.e. documents) Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in populations. The question is framed so that the response will allow for the differences. Questions can result in a narrow or wide range of numerical values. <u>Example:</u> Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking “Do you exercise?” they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “ How many hours per week on average do students at Teeland Middle School exercise?”
6.SP.2 Understand that a set of data has a distribution that can be described by its center (mean, median, or mode), spread (range), and overall shape and can be used to answer a statistical question.	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shap of the distribution of the data. Students examine the distribution of a data set and discuss the center ,spread and overall shape with dot plots, histograms and box plots <u>Example 1:</u>

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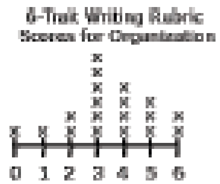

The dot plot shows the writing scores for a group of students on organization. Describe the data.
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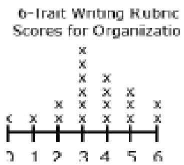
<td>6.SP.3 Recognize that a measure of center (mean, median, or mode) for a numerical data set summarize all of its values with a single number, while a measure of a variation (range) describes how its values vary with a single number.</td>
<td>
Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie.midpoint of an ordered list or rhe balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measured of variability are used to describe this characteristic.

Example 1:
Consider the data shown in the dot plot of the six trit scores for organization for a group of students.
* How many student are represented in the data set?
* What are the mean and median of the data set? What do these values mean? How do they compare?
*What is the range of the data? What does this value mean?



Solution:
* 19 students are represented in the data set
.* The mean of the data set is 3.5. The median is 3 The mean indicates that is the values were equally distributed, all student would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
* The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.
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<td>6. SP.4 Display numerical data in plots on a number line, including dot or line plots, histograms and box (box-and-whisker plots)</td>
<td>
Students display data graphically using number lines. Dot plots, histograms, and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.
Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either
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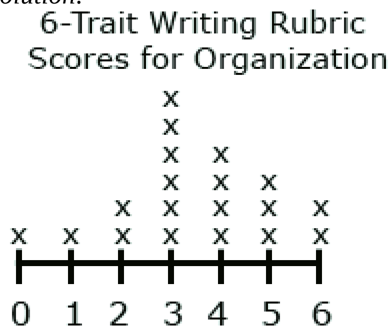
vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.

Example 1:
Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

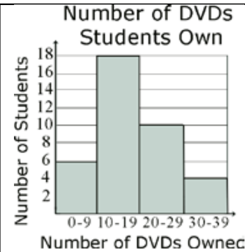
Solution:



Example 2:
Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:
A histogram using 5 intervals (bins) 0-9, 10-19, …30-39) to organize the data is displayed below



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:

Five number summary

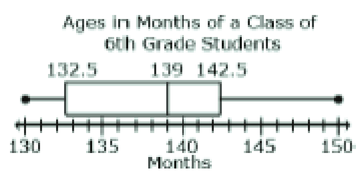
Minimum – 130 months

Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months

Median (Q2) – 139 months

Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months

Maximum – 150 months



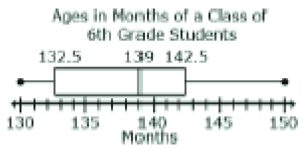
This box plot shows that

- □□□ of the students in the class are from 130 to 132.5 months old
- □□□ of the students in the class are from 142.5 months to 150 months old
- □□□□ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

6.SP.5 Summarize numerical data sets in relation	Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random

to their context, such as by: a. Reporting the number of observations(occurrences) b. Describing the nature of the attribute under investigation, including how it was measured and its unit measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any outliers with reference to the context in which the data were gathered. d. Relating the choice of measure of center and variability to the shape of the data distribution and the context in which the data were gathered.	sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles. Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). <u>Measures of Center</u> Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point). Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average. Example 1: Susan has four 20-point projects for math class. Susan’s scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 Project 4: ?? What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning. *Solution:* One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19). <u>Measures of Variability</u> Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers. Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:
What is the IQR of the data below:



Solution:
The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations.
Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2:
The following data set represents the size of 9 families:
3, 2, 4, 2, 9, 8, 2, 11, 4.
What is the MAD for this data set?

Solution:
The mean is 5. The MAD is the average variability of the data set. To find the MAD:
1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

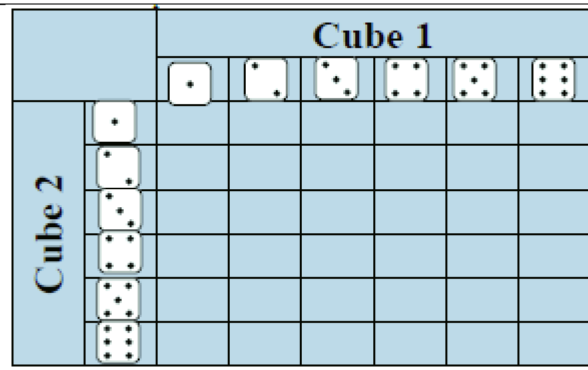
The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
	MAD	26/9 = 2.89

This value indicates that on average family size varies 2.89 from the mean of 5.

Students understand how the measures of center and measures of variability are represented by graphical displays.

	Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.
6.SP.6 Analyze whether a game is mathematically fair or unfair by explaining the probability of all possible outcomes.	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? Example 1: Suppose you are approached by a classmate who invites you to play a game with the following rules: Each of you takes a turn flipping a coin. You toss your coin first, and he tosses his coin second. • He gives you $1 each time one of the coins lands on tails. • You give him $1 each time one of the coins land on heads. a. Create a tree diagram for the four possible outcomes and probabilities for the two tosses. b. List all of the possible outcomes. c. What are your winnings for each outcome?
6.SP.7 Solve or identify solutions to problems involving possible combinations (e.g., if ice cream sundaes come in 3 flavors with 2 possible topping, how many different sundaes can be made using only one flavor of ice cream with one topping?)	Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. Example 1: **Part 1** You roll a pair of fair six-sided number cube and find the sum of the uppermost faces. 1. What are all of the possible outcomes? Fill in the chart below.



2. How many total outcomes are possible?

3. What is the probability of rolling a sum of 6?

4. What sums have the smallest probability?

Part Two

Suppose you roll two number cubes.

5. Make a table to show all of the possible outcomes (use an another piece of paper)

6. What is the probability that you will roll doubles?

7. Are “rolling a sum of 6” and “rolling doubles” equally likely events? Justify your answer.